

1. (14 points) Let X and Y have a joint probability density function given by

$$f(x, y) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq y \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Compute the marginal probability density functions for X and Y .
- (b) Compute $P(X \leq 1, Y \leq \frac{1}{2})$.
2. (12 points) Suppose X_1 and X_2 are independent Exponential (λ) random variables. Find the conditional distribution of X_1 given $X_1 + X_2 = z$ for some $z > 0$.
3. (14 points) Let X, Y, Z be independent random variables that each have the uniform distribution on the interval $(0, 1)$. Find $P\left(\frac{X}{Y} > \frac{Y}{Z}\right)$.
4. (15 points) Let $X \stackrel{d}{=} N(0, 1)$ and Z be an independent random variable such that $P(Z = 1) = P(Z = -1) = \frac{1}{2}$.
- (a) Let $Y = ZX$. Find the probability density function of Y .
- (b) Suppose X_1, X_2 are two Normal random variables with mean 0, variance 1 and covariance 0. Decide when does it imply that X_1 and X_2 are independent random variables.